Quantum coherence and entanglement in the avian compass: Supplementary Material

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[1]

FIG. 1: Results for a disc-shaped HF tensor. The graph on the left is precisely analogous to Fig. 2 of the main text, showing angular dependence of the singlet yield in the presence of the oscillatory eld for di erent decay rates k. The graph on the right corresponds to Fig. 3 of the main paper, showing angular dependence of the singlet yield in the presence of environmental

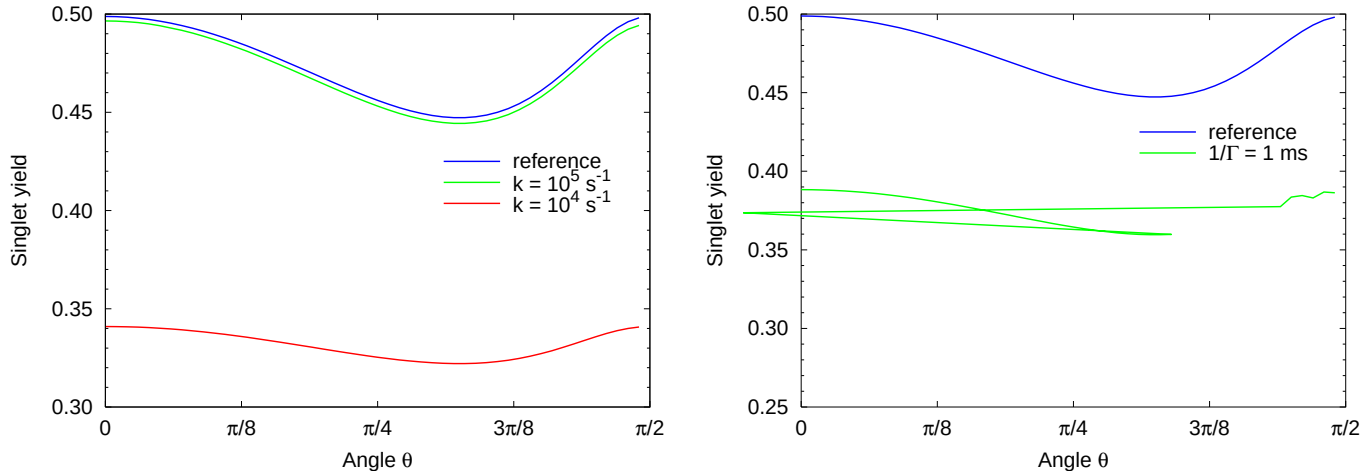

FIG. 2: Results for a model in which an anisotropic g-factor replaces the role of the nucleus to break the RP symmetry. The graph on the left is precisely analogous to Fig. 2 of the main text, showing angular dependence of the singlet yield in the presence of the oscillatory eld for di erent decay rates k. The graph on the right corresponds to Fig. 3 of the main paper, showing angular dependence of the singlet yield in the presence of environmental noise for di erent noise rates . Although the curve shapes di er, remarkably the levels of contrast remain similar to those of the conventional model in the main paper.

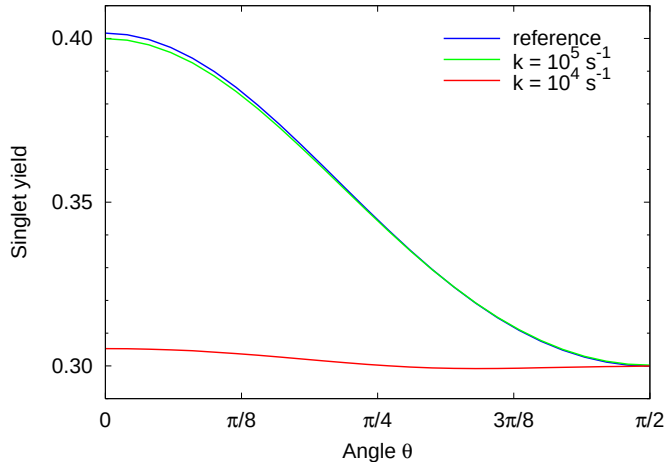

FIG. 3: Results for two nuclear spins coupled to electron 1. The graph on the left is precisely analogous to Fig. 2 of the main text, showing angular dependence of the singlet yield in the presence of the oscillatory eld for di erent decay rates k. The graph on the right corresponds to Fig. 3 of the main paper, showing angular dependence of the singlet yield in the presence of environmental noise for di erent noise rates . Although the curve shapes di er somewhat, nevertheless the parameters corresponding to serve degradation remain the same as those in the main paper ($k = 10^4$ s ^{1}, = 100 s).

but gives less overall signal contrast (even in the unperturbed) scenario. We present results for these parameters in Fig. 2, con rming that the qualitative behaviour is still the same under these assumptions.

RP pair model with two nuclear spins

In the model described in the main paper, there is a single nuclear spin coupled to one of the electrons. However, previous publications have studied the case where more than one nuclear spin is present [1, 3, 4]. Therefore, it is interesting to check whether the addition of a nuclear spin will alter our conclusions. The Hamiltonian now gains an additional coupling term,

$$H = \hat{I}_1 \; A_1 \; \hat{S}_1 + \hat{I}_2 \; A_2 \; \hat{S}_1 + \; B \; (\hat{S}_1 + \hat{S}_2);$$

where A_1 is the HF tensor of the main text and $A_2 = 2=3A_1$. Here we choose a second rugby shaped HF tensor oriented parallel to the rst. Note that we have also considered di erent relative coupling strengths and geometries (such as a pancake shaped and rugby shaped tensor), but again, these choices do not in uence our core conclusions. Results for the particular choice of parameters described above are shown in Fig. 3.

PURE DEPHASING NOISE MODEL

Interestingly, if we begin the simulation with a completely dephased state: $(jsihsj + jt_0iht_0j)=2$, the classical correlations are still su cient for achieving adequate angular visibility and neither quantum phase coherence nor entanglement seems to be a prerequisite for the e ciency of the avian compass.

To explore this idea further, we would like to study `pure dephasing' occurring *during* the singlet-triplet interconversion. In essence we use energy conserving noise operators, Eqn. (1), which are known to be the dominant source of decoherence in so many other arti cially made quantum systems. By applying this speci c noise, we con rm that the compass mechanism's performance is essentially immune, while of course the coherence of the quantum state of the electrons would be degraded.

One might be inclined to conclude that, if pure dephasing noise is indeed dominant, then the avian compass need not protect quantum coherence for the long time scales suggested in the main paper. But crucially, we also show that if such noise were naturally present at a high level in the compass (exceeding the generic noise level by more than an order of magnitude) then it would render the bird immune to the weak oscillatory magnetic elds studied by Ritz *et al.* [1]. Thus the sensitivity to oscillatory elds implies that both amplitude and phase, and thus entanglement, are indeed protected within the avian compass on timescales exceeding tens of microseconds.

Since the electron spin singlet state is not an eigenstate of the Hamiltonian, the dephasing operators will be di erent from the ones mixing the phase of the singlet and triplet state within the electronic subspace. Instead, we replace the previously de ned noise operators of L_i of Eq. (3) by appropriate dephasing operators as follows: we treat the remote electron and the electron nuclear spin subsystem separately. Within both subsystems, we de ne dephasing operators

$$Z_i = \frac{1}{\sqrt{2}}\left(\sum_{j\neq i} |j\rangle\langle j| - |j\rangle\langle i|\,\langle i|\langle j|\right) = \frac{1}{\sqrt{2}}(I_4 - 2|i\rangle\langle i|); \qquad (1)$$

where $\{|i\rangle\}$ are the set of normalised eigenvectors of this subsystem. This results in two dephasing operators for the remote electron (these can be combined to a single $_z$ operator rotated with the eld) and four operators for the electron nuclear spin subsystem. Each of these dephasing operators corresponds to uctuations of one of the (subsystem's) energy levels.

Strikingly, the singlet yield is entirely una ected by this particular kind of noise, i.e. it is entirely independent of the dephasing rate $_z$. Thus, a curve obtained with this model coincides perfectly with the reference curve of Fig. 3 of the main paper. However, we show in the following that the dephasing rate of this model can be at most ten times faster than the generic noise rate to retain sensitivity to the oscillatory eld.

Fig. 4 shows the singlet yield as a function of for di erent pure dephasing rates $_z$. Pure phase noise would actually *protect* the compass from the harmful e ect of an applied oscillatory eld (by suppressing the Rabi oscillations caused by such a eld). We see that an aggressive pure dephasing rate of $1=_z = 10$ s almost completely recovers the reference curve (corresponding to a noise-free system without oscillatory eld).

above with the `dotted line picture' some level of Markovian noise will also be inevitably present on the long timescale required for the RP process, thus we are still dealing with a computationally demanding open systems problem, limiting the number of spins that can be simulated.

Figure 6 shows results obtained for a RP model consisting of two electron and four nuclear spins. As before, one of the nuclear spins is a central part of the mechanism through its anisotropic coupling to electron 1 with parameters as in the main paper. The remaining three nuclear spins serve as an additional local spin bath, and we have assumed isotropic hyper ne coupling strengths of $A2 = A_z$=8; $A3 = A_z$=10; $A3 = A_z$=12 with electron 1 ($A_z = 10^{-5}$ meV). All nuclear spins are initialised in a fully mixed state as appropriate for a room temperature environment. The presence of the additional nuclear spins introduces some ripples into the shape of the curves, yet there is no qualitative di erence with regard to the e ects of irreversible noise and the oscillatory eld. This means that we must once more arrive at the same conclusion that the compass system is remarkably well protected from an irreversible loss of coherence. We note that further calculations with only three protons and for a range of di erent coupling strengths (not shown) display the same qualitative behaviour, indicating that there is no trend for an increasing number of spins that would alter our conclusion.

ADDITIONAL NOISE MODELS

Having simultaneously considered all 63 processes, we now look at the e ect of individual Lindblad operators. We focus on the nine subsets of operators which a ect coherence on *both* electron spins (otherwise the coherence time of at least one part of the compass would be in nite). Summing over the four possible nuclear combinations, we obtain the manageable number of nine noise models, each with isotropic noise on the nuclear spin [5].

All curves in our calculations have the same shape with the largest singlet yield achieved for = 0 and the lowest singlet yield in the perpendicular con guration = =2. The di erence in singlet yield between those two con gurations corresponds to the signal contrast and is a good measure for judging the performance of the compass. In the absence of noise and the oscillatory eld, the contrast is 0:382 0:281 = 0:101, and we shall use this value as the benchmark for comparing the di erent noise models.

Noise model	Noise [%]	Noise & RF [%]	Degradation [%]	Noise [%]	Noise & RF [%]	Degradation [%]
	1= = 100 s			1= = 10 s		
general noise	14.49	13.98	3.54	1.65	1.649	0.05
$_x \otimes _x$	39.09	34.55	11.62	5.85	5.83	0.23
$_x \otimes _y$	40.45	35.78	11.55	6.05	6.03	0.23
$_x \otimes _z$	12.72	11.61	8.73	1.42	1.41	0.16
$_y \otimes _x$	40.45	35.82	11.46	6.05	6.03	0.23
$_y \otimes _y$	39.09	34.59	11.51	5.84	5.83	0.23
$_y \otimes _z$	11.36	10.42	8.26	1.21	1.21	0.16
$_z \otimes _x$	12.72	12.24	3.77	1.42	1.42	0.05
$_z \otimes _y$	11.36	11.02	3.02	1.21	1.21	0.03
$_z \otimes _z$	39.09	35.98	7.97	5.84	5.83	0.15
pure dephasing	99.999	63.45	36.55	99.991	92.41	7.58
match experiment	high	strongly reduced	high	high	strongly reduced	high

TABLE I: Contrast for the nine di erent Lindblad operators compared to the isotropic noise model of the main paper and the pure dephasing noise. The \Noise model" column speci es the type of noise and for the $_i \otimes _j$ entries $_i$ and $_i$ are the noise acting on the `local' and `remote' electron spin respectively. The \Noise" and \Noise & RF" columns give the signal contrast as a percentage of the ideal unperturbed system (see text). The \ucm141..855 401..855 44 J 0.398 w 0 0 m 490.837 0 l S Q BT /F15

[2] I.A. Solov'yov, D. E. Chandler, K. Schulten:*Magnetic Field E ects in Arabidopsis thaliana Cryptochrome-1*, Biophysical Journal, 92, 2711-2726 (2007).
[3] C. T. Rodgers, P.J. Hore: *Chemical magnetoreception in birds: The radical pair mechanism*, PNAS, 106 2, 353-360, (2009).
[4] C.T. Rodgers:*Magnetic Field E ects in Chemical Systems*, PhD Thesis, Oxford (2007).
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